CPSC 311: Analysis of Algorithms
Proof by Induction Example Problems

These problems are solely for your own benefit in practicing inductive proofs. They will not be turned in or graded. A solution sheet will not be handed out. If you want to discuss whether you are doing them correctly, you can meet with the instructor.

Prove the following statements using induction.

References: *A Logical Approach to Discrete Math* by David Gries and Fred B. Schneider, Springer-Verlag, 1994, and *The Art of Computer Programming: Vol 1* by Donald Knuth, Addison-Wesley, 1973.

1. For all $n \geq 4$, there exist nonnegative integers h and k such that $2 \cdot h + 5 \cdot k = n$.
2. For all $n \geq 0$, $\Sigma_{i=1}^{n} i = n \cdot (n+1)/2$.
3. For all $n \geq 0$, $\Sigma_{i=0}^{n-1}(2 \cdot i + 1) = n^2$.
4. For all $n \geq 0$, $\Sigma_{i=0}^{n-1} 3^i = (3^n - 1)/2$.
5. For all $n \geq 0$, $2^{2n} - 1$ is divisible by 3.
6. Define the value $n!$ recursively for all $n \geq 0$ as follows.
   Let $0! = 1$ and let $n! = n \cdot (n-1)!$ for all $n > 0$. Prove that $n! = 1 \cdot 2 \cdot \ldots \cdot (n-1) \cdot n$.
7. Prove that $n! > 2^{n-1}$ for all $n \geq 3$.
8. Define a sequence of numbers S_n recursively for all $n \geq 0$ as follows.
   Let $S_0 = 0$. Let $S_n = 2 \cdot S_{n-1} + 1$ for all $n > 0$. Prove that $S_n = 2^n - 1$ for all $n \geq 0$.
9. What is wrong with the following 'proof'?

   Claim: Let a be any positive number. For all positive integers n, we have $a^{n-1} = 1$.

   Proof:
   Basis: If $n = 1$, $a^{n-1} = a^{1-1} = a^0 = 1$.
   Inductive Hypothesis: Assume $a^{n-1} = 1$ for all $n = 1, 2, \ldots, k$.
   We now show that it is true for $n = k + 1$:

   $$a^{(k+1)-1} = a^k = \frac{a^{k-1} \cdot a^{k-1}}{a^{k-2}} = \frac{1 \cdot 1}{1} = 1.$$

   The inductive hypothesis was used to replace a^{k-1} and a^{k-2} by 1.

   □